1/

1991 PAPER 2

#1 (a) $f''(x) = 24x - 18$

$f'(x) = \frac{24x^2}{2} - 18x + c = 12x^2 - 18x + c$

$f'(1) = -6 \Rightarrow -6 = 12(1)^2 - 18(1) + c$

$-6 = 12 - 18 + c$

$c = 0$

$\Rightarrow f'(x) = 12x^2 - 18x$

when $f'(x) = 0$, $6x(2x-3) = 0$ $\boxed{\begin{array}{l} x = 0 \\ x = 3/2 \end{array}}$

(b) $f(x) = \int f'(x)\,dx = \frac{12x^3}{3} - \frac{18x^2}{2} + A = 4x^3 - 9x^2 + A.$

$f(2) = 0 \Rightarrow 0 = \frac{96}{3} - 9(2)^2 + A$

$0 = 32 - 36 + A$

$A = 4$

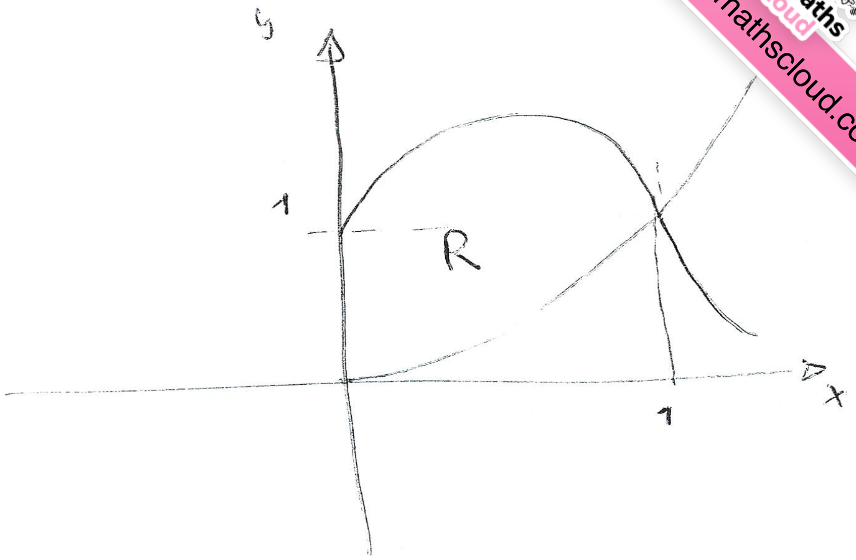
$\Rightarrow \boxed{f(x) = 4x^3 - 9x^2 + 4}$

(c) $f(x)_{av} = \frac{\int_1^3 f(x)\,dx}{3-1} = \frac{\int_1^3 4x^3 - 9x^2 + 4\,dx}{2}$

$= \frac{\left(x^4 - 3x^3 + 4x\right)\Big|_1^3}{2} = \frac{12-2}{2} = \boxed{5}$ Ans:

#2 $y = 1 + \sin \pi x$

$y = x^2$



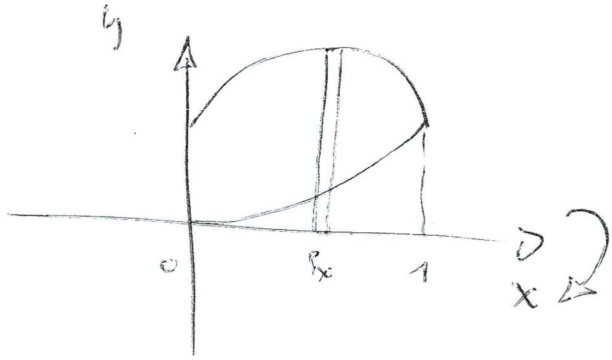
(a) $A = \int_0^1 1 + \sin \pi x - x^2 \, dx$

$A = \left(x - \frac{\cos \pi x}{\pi} - \frac{x^3}{3} \right) \Big|_0^1$

$A = 1 - \frac{\cos \pi}{\pi} - \frac{1}{3} + \frac{1}{\pi} = \frac{2}{3} + \frac{2}{\pi} = \frac{2\pi + 6}{3\pi} = \boxed{\frac{2(\pi+3)}{3\pi}}$

(b) $V = \pi \int_0^1 (1 + \sin \pi x)^2 - x^4 \, dx$

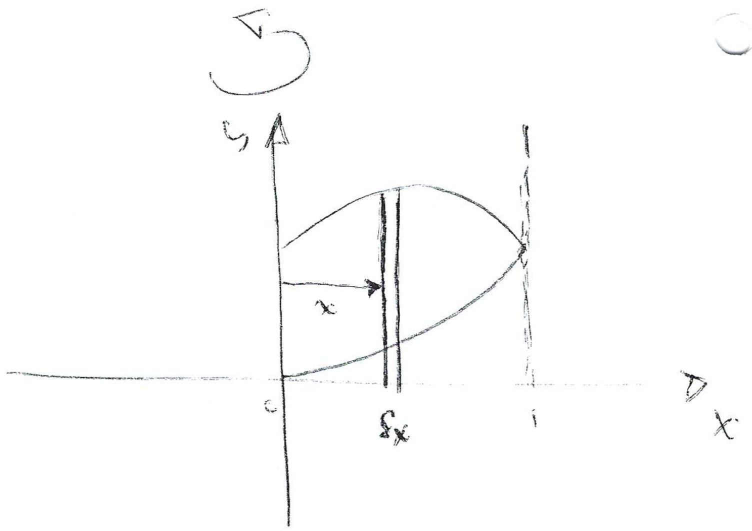
(disc)



(c) (shell)

$V = 2\pi \int_0^1 xy \, dx$

$V = 2\pi \int_0^1 x(1 + \sin \pi x - x^2) \, dx$



3 $\quad f(x) = (1+\tan x)^{3/2}$

$$\boxed{-\frac{\pi}{4} < x < \frac{\pi}{2}}$$

(a) $f'(x) = \frac{3}{2}(1+\tan x)^{1/2} \cdot \sec^2 x$

$f'(0) = \frac{3}{2}$ $\qquad\qquad x=0,\ f(0)=1$

$\Rightarrow \quad y - y_1 = m(x - x_1)$

$y - 1 = \frac{3}{2}(x - 0)$

$2y - 2 = 3x$

$$\boxed{3x - 2y + 2 = 0}$$

(b) The tangent at $(0,1)$ approximates the function at that point.

$f(0.02) \simeq \frac{3}{2}(0.02) + 1 \simeq \boxed{1.03}$

(c) $y = (1+\tan x)^{3/2}$

$x = (1+\tan y)^{3/2}$

$x^{2/3} = 1 + \tan y$

$\tan y = x^{2/3} - 1$

$y = \tan^{-1}(x^{2/3} - 1)$

$$\boxed{f^{-1} : x \mapsto \tan^{-1}(x^{2/3} - 1)}$$

NB The domain of f^{-1} is the range of f.

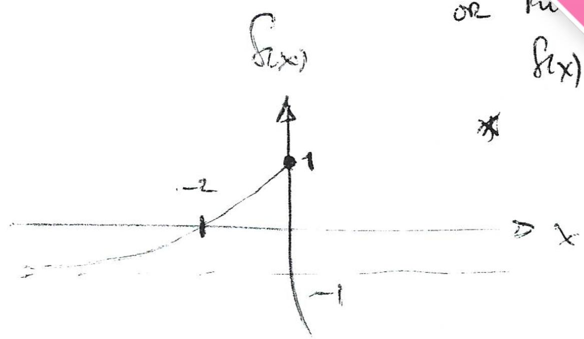
Domain: $\{0 < x < \infty\}$

#4 $\quad f(x) = \dfrac{|x|-2}{x-2}$

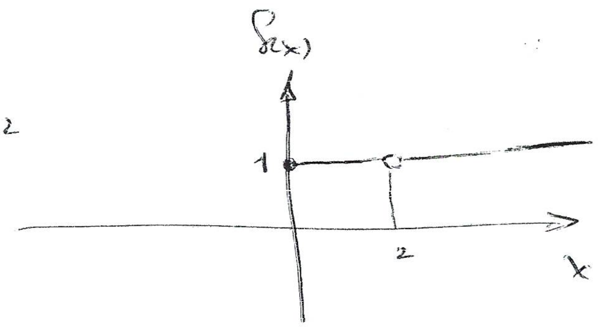
or fu...

$f(x)=$...

$x<0 \qquad f(x) = -\dfrac{x-2}{x-2}$



$x \geq 0 \qquad f(x) = \dfrac{x-2}{x-2} = 1 \qquad x \neq 2$



(a) $x = -2$ $\qquad$ (b) zero

(c) $f'(x) = \dfrac{(x-2)(-1) + x+2}{(x-2)^2}$ $\qquad$ for $x<0$

$f'(x) = \dfrac{4}{(x-2)^2}$

$\boxed{f'(-1) = 4/9}$

(d) Range of $f \qquad (-1, 1]$

(a) $6\begin{cases} 2\ f(x) & x< \\ 2\ f(x) & x\geq \\ 1\ f(x)=0 \\ 1\ \text{soln} \end{cases}$

(b) $2\begin{cases} 1:\ f' \\ 1:\ \text{soln} \end{cases}$

(c) $3\begin{cases} 2\ f(x) \\ 1:\ \text{soln} \end{cases}$

(d) $4\begin{cases} 2:\ \text{graph} \\ 2:\ \text{soln} \end{cases}$

#5 (a) $x=0$, f attains absolute maximum.

$x=\pm 2$, f attains absolute minima.

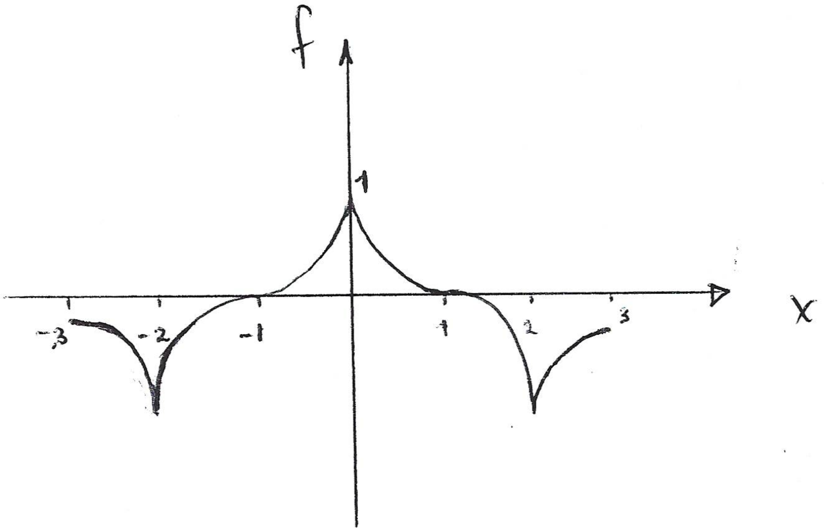
(b) $x=\pm 1$

	$0<x<1$	$x=1$	$1<x<2$
$f'(x)$	$-$	0	$-$

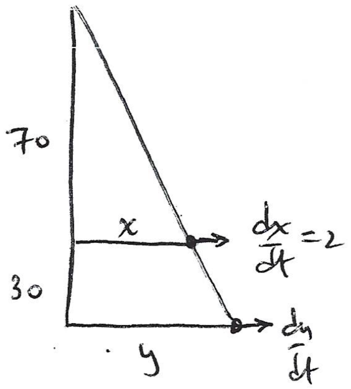
Horizontal Points of Inflexion.

	$-2<x<-1$	$x=-1$	$-1<x<0$
$f'(x)$	$-$	0	$-$

(c)



#6 (a)

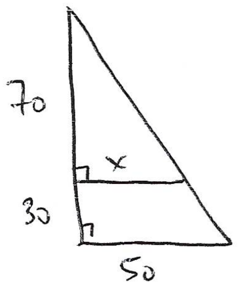


By similar △'s:

$$\frac{70}{x} = \frac{100}{y} \Rightarrow y = \frac{100x}{70} = \frac{10x}{7}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{10}{7} \cdot 2 = \boxed{\frac{20}{7}\text{ ft/s}}$$

(b)

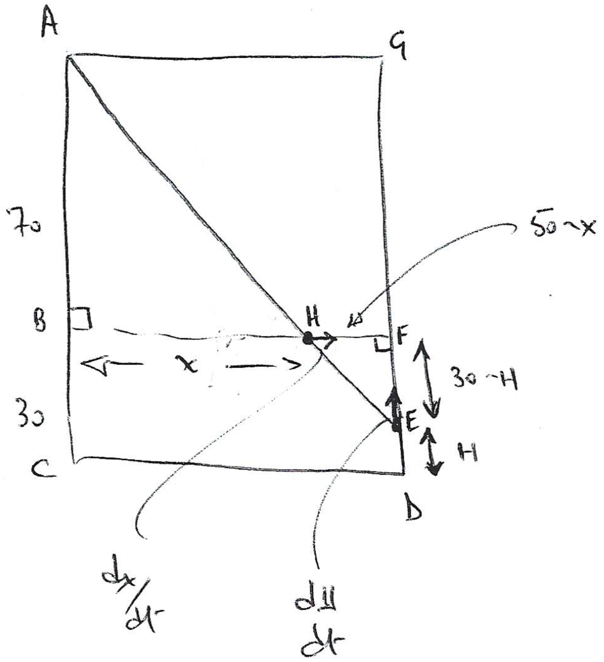


$$\frac{70}{x} = \frac{100}{50} = \frac{2}{1}$$

By Similar △'s:

$$\boxed{x = 35\text{ ft}}$$

(c)



W.K.O

△ ABG ~ △

$$\frac{70}{x} = \frac{30-H}{50-x}$$

$$70(50-x) = (30-H)x$$

$$30 - H = \frac{3500 - 70x}{x}$$

$$H = 70 - \frac{3500}{x} + 30 \qquad x \neq 0$$

$$H = 100 - 3500x^{-1}$$

$$\frac{dH}{dx} = \frac{3500}{x^2}$$

(when $x = 40$)

$$\Rightarrow \frac{dH}{dt} = \frac{dH}{dx} \cdot \frac{dx}{dt} = 3500 \cdot 2 = 3500 \cdot 2 = 70 = \boxed{4\tfrac{3}{8}\text{ ft/s}}$$